HW8 solutions

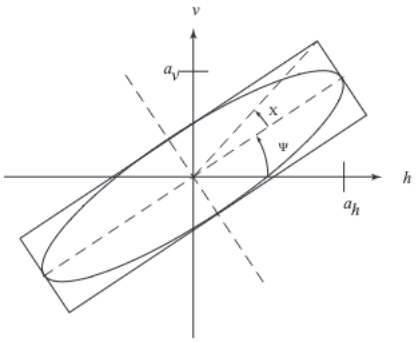
We have scattering matrix

$$F = \begin{bmatrix} I \\ I\cos 2\psi \cos 2\chi \\ I\sin 2\psi \cos 2\chi \\ I\sin 2\chi \end{bmatrix}$$

Note that the perpendicular scattering matrix

$$F_{\perp} = \begin{bmatrix} I \\ -I\cos 2\psi \cos 2\chi \\ -I\sin 2\psi \cos 2\chi \\ -I\sin 2\chi \end{bmatrix}$$

How do we get this?
From the polarization ellipse. Let $\psi_{\perp} = \psi + 90°$ and $\chi_{\perp} = -\chi$:



Problem 1: Dihedral corner reflector

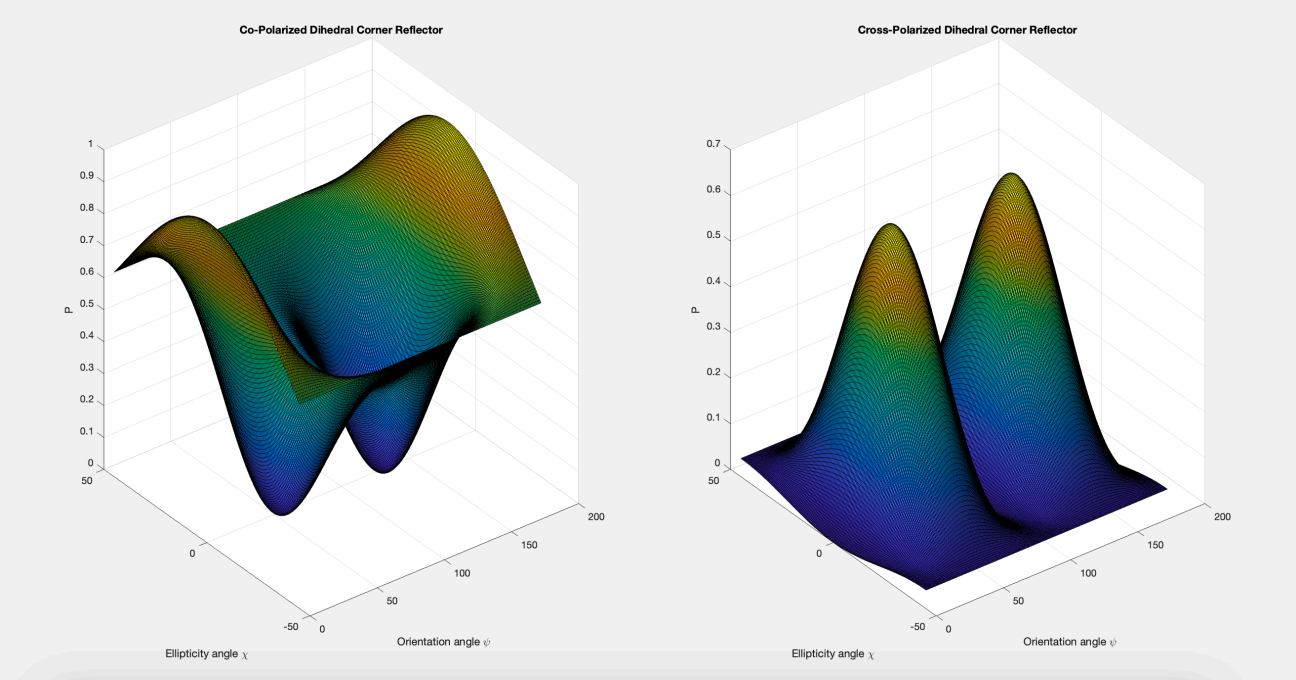
a. No background

We know that our polarization matrix

$$\begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0.6 \end{bmatrix}$$

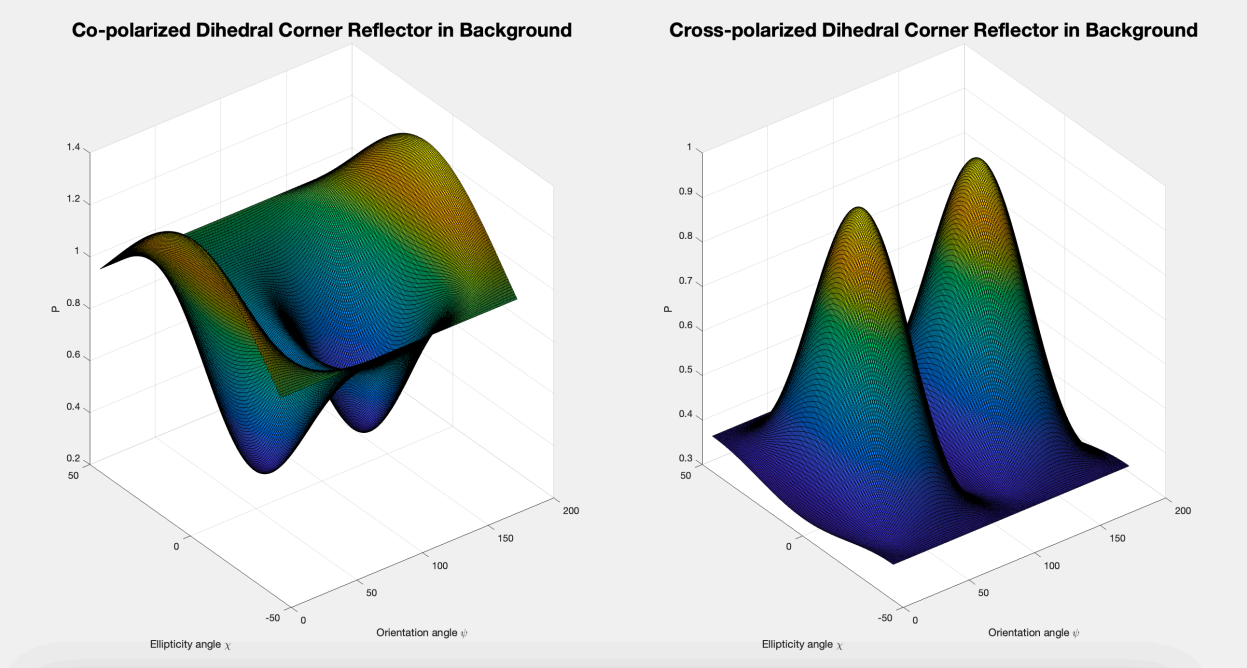
So we substitute Shh = -1 and Svv = 0.6 into the Stokes Matrix according to Table 16.2.

Then we cycle through χ, ψ and find $P_{co} = F'MF, P_{cross} = F'MF_{\perp}$ for all values of chi and psi, and make a surface plot.



b. Background

Adding the background means we make a new scattering matrix with only one element: the (1,1) element. That element represents total power; because that power has no particular polarization, all other elements are 0.

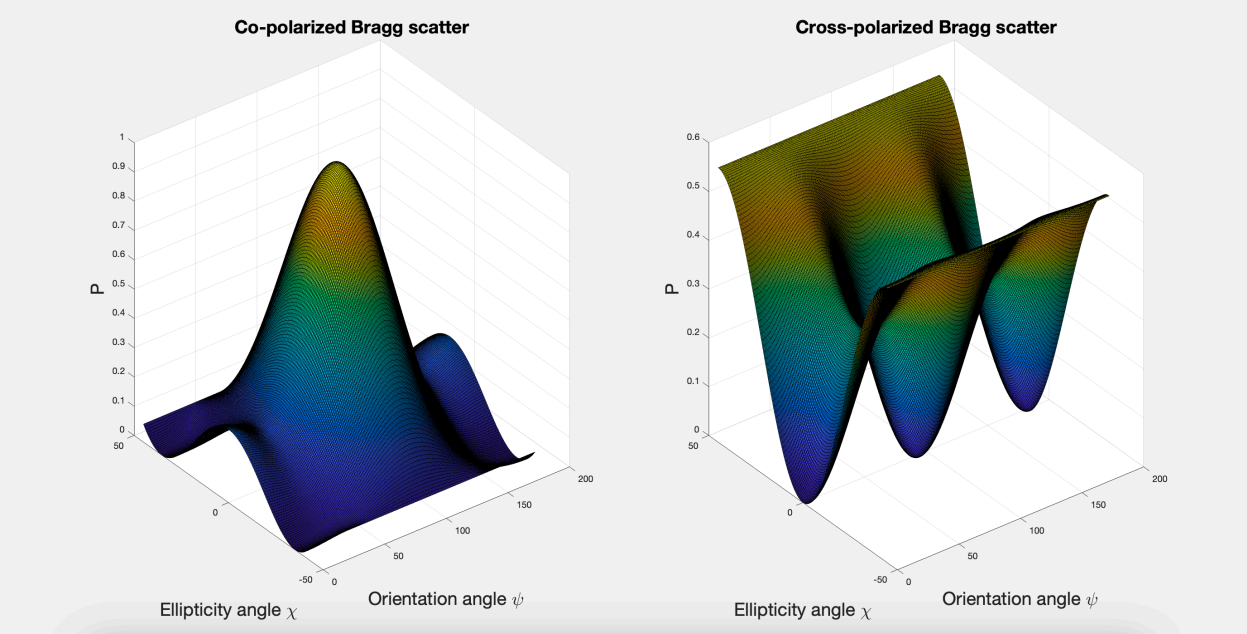


The background kinda just universally raises the signal up.

Problem 2: Bragg model

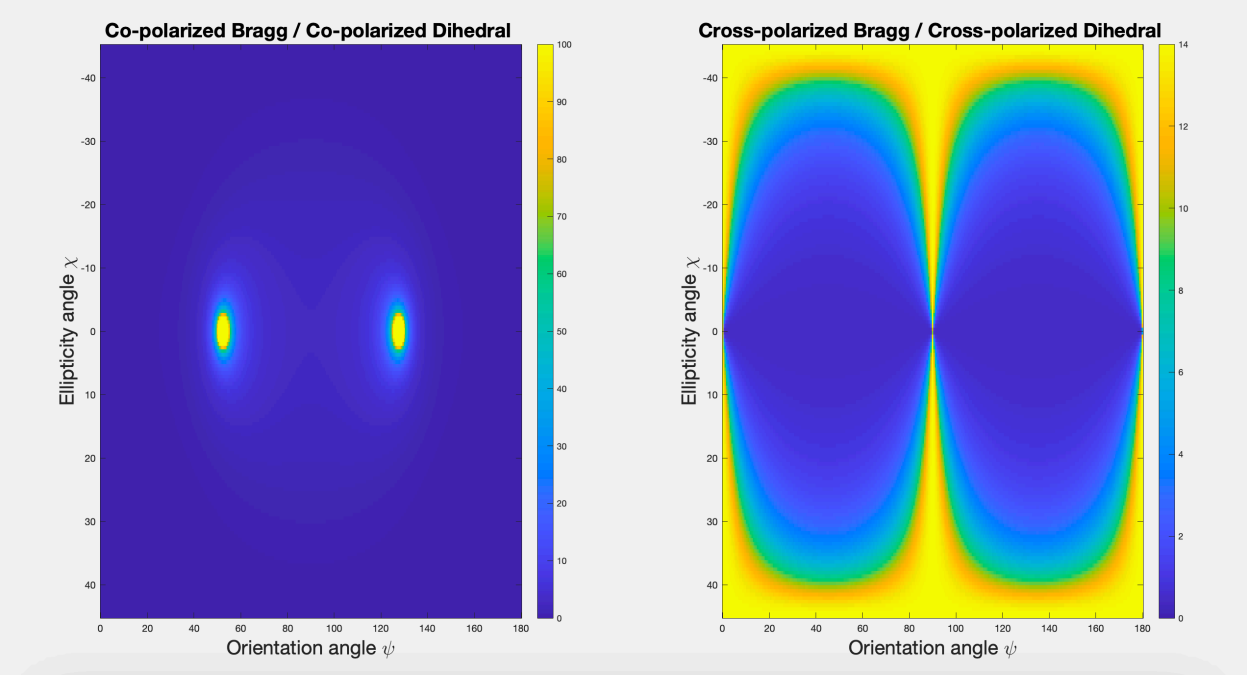
We now find a new version of M with Shh = 0.5, Svv = 1.

a. Cross-and co-polarized, and comparison to dihedral

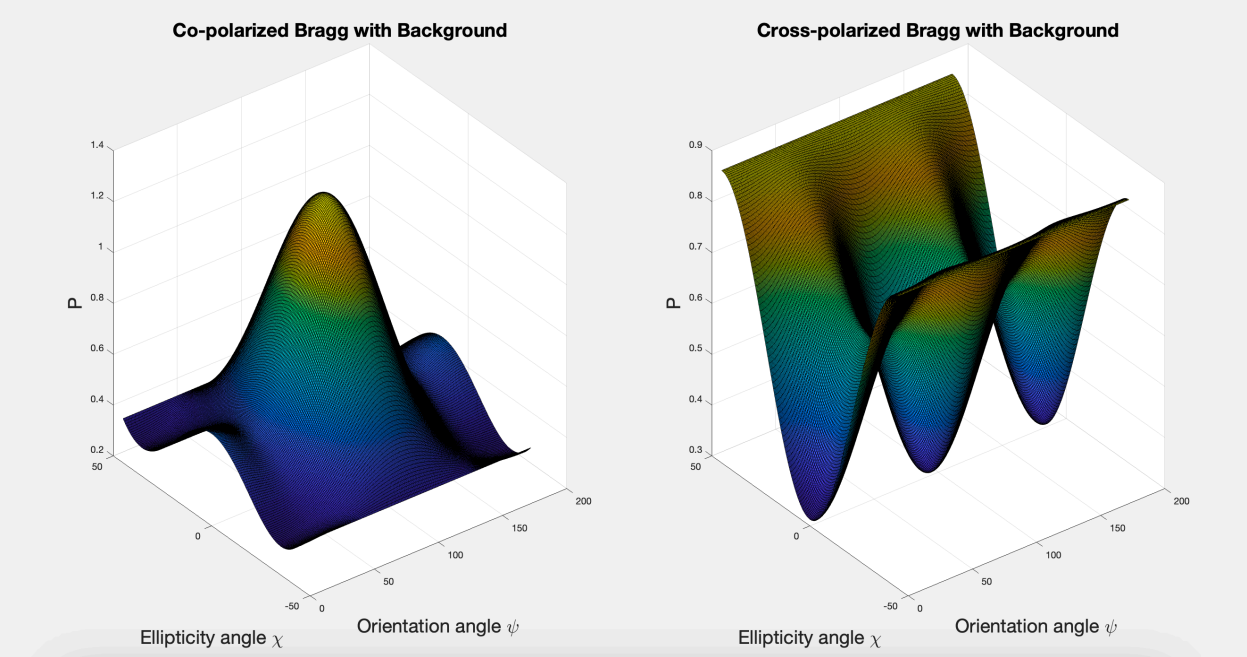


Above: Co- and cross-polarized Bragg signatures.

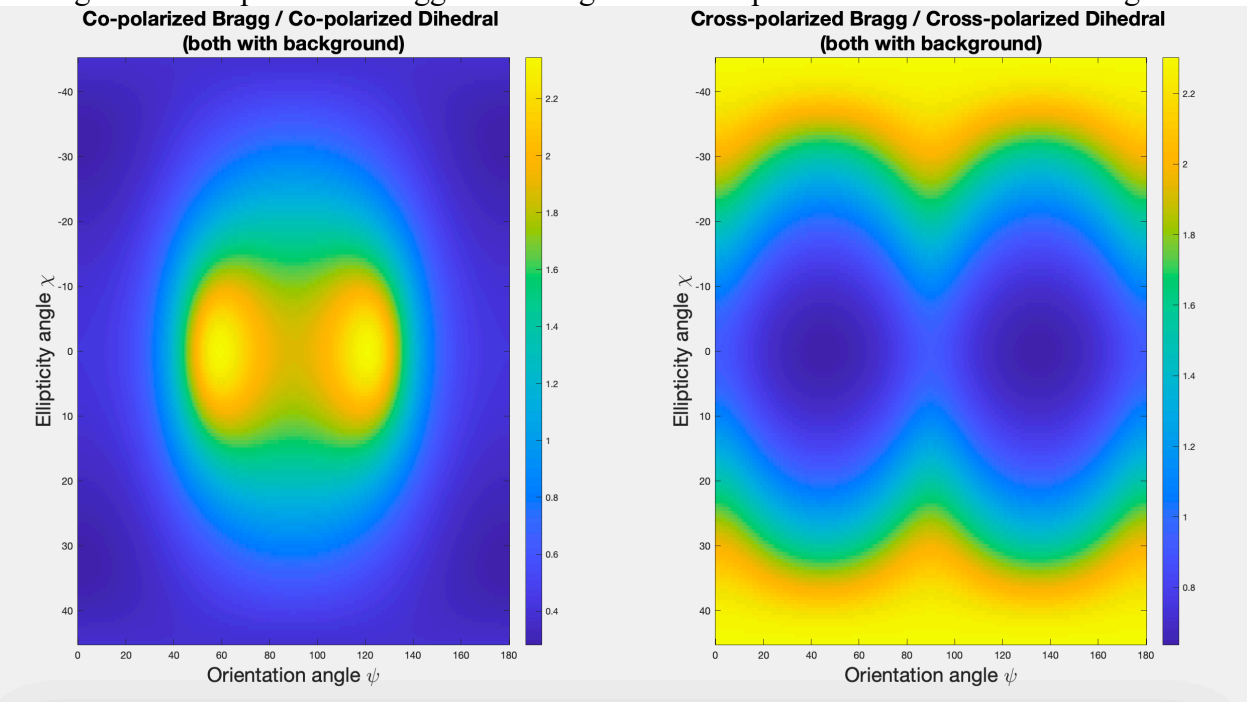
Taking the ratio of this to the dihedral:



b. Plus background, and compare to dihedral with background



Taking ratio of co-polarized Bragg with background to co-polarized Dihedral with background:



MATLAB code:

```
% Elizabeth Wig
% NISAR/JPL Radar Course HW 8 Polarization
% Problem 1
% Part a
chi = -pi/4:.01:pi/4;
psi = 0:0.01:pi;
% [Chi, Psi] = meshgrid(chi,psi);
chi_orth = -chi;
psi_orth = psi + pi/2;
I = 1;

a = 1;
b = 0.6;
S = [-a, 0; 0, b];
Shh = -a;
Shv = 0;
Svh = 0;
Svv = b;
M1a = [1/4*(Shh*conj(Shh)+Svv*conj(Svv)+2*Shv*conj(Shv)), ... % 1,1
    1/4 *(Shh*conj(Shh)-Svv*conj(Svv)),... % 1,2
    1/2*(real(conj(Shh)*Shv)+real(conj(Shv)*Svv)),... %1,3
    1/2*(imag(conj(Shh)*Shv)+real(imag(Shv)*Svv));... %1,4
    1/4*(Shh*conj(Shh)-Svv*conj(Svv)),...%2,1
    1/4*(Shh*conj(Shh)+Svv*conj(Svv)-2*Shv*conj(Shv)), ... %2,2
    1/2*(real(conj(Shh)*Shv)-real(conj(Shv)*Svv)),... % 2,3
    1/2*(imag(conj(Shh)*Shv)-real(imag(Shv)*Svv));...% 2,4
    1/2*(real(conj(Shh)*Shv)+real(conj(Shv)*Svv)),... %3,1
    1/2*(real(conj(Shh)*Shv)-real(conj(Shv)*Svv)),... % 3,2
    1/2*(Shv*conj(Shv)+real(conj(Shh)*Svv)), ... % 3,3
    1/2*imag(conj(Shh)*Svv); ... %3,4
    1/2*(imag(conj(Shh)*Shv)+real(imag(Shv)*Svv)),... %1,4
    1/2*(imag(conj(Shh)*Shv)-real(imag(Shv)*Svv)),...% 2,4
    1/2*imag(conj(Shh)*Svv), ... %3,4
    1/2*(Shv*conj(Shv) - real(conj(Shh)*Svv))... %4,4
    ];

P1a = zeros(length(chi),length(psi));
for m = 1:length(chi)
    for n = 1:length(psi)
        F = [I; I*cos(2*psi(n))*cos(2*chi(m));
I*sin(2*psi(n))*cos(2*chi(m)); I*sin(2*chi(m))];
%         Forth = [I; I*cos(2*psi_orth(n))*cos(2*chi_orth(m));
I*sin(2*psi_orth(n))*cos(2*chi_orth(m)); I*sin(2*chi_orth(m))];
        Forth = [F(1); -F(2); -F(3); -F(4)];
        P1a(m,n) = F'*M1a*F;
        P1across(m,n) = F'*M1a*Forth;
    end
end

figure;
subplot(1,2,1); surf(psi*180/pi, chi*180/pi, P1a);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P');
```

```
title('Co-Polarized Dihedral Corner Reflector','FontSize',20)

subplot(1,2,2); surf(psi*180/pi, chi*180/pi, P1across);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P');
title('Cross-Polarized Dihedral Corner Reflector','FontSize',20)


%% Part b
M1b = M1a;
M1b(1,1) = 2*M1a(1,1);
P1b = zeros(length(chi),length(psi));
for m = 1:length(chi)
    for n = 1:length(psi)
        F = [I; I*cos(2*psi(n))*cos(2*chi(m));
I*sin(2*psi(n))*cos(2*chi(m)); I*sin(2*chi(m))];
        Forth = [F(1); -F(2); -F(3); -F(4)];
        P1b(m,n) = F'*M1b*F;
        P1bcross(m,n) = F'*M1b*Forth;
    end
end
figure; subplot(1,2,1);
surf(psi*180/pi, chi*180/pi, P1b);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Co-polarized Dihedral Corner Reflector in Background','FontSize',20)
subplot(1,2,2);
surf(psi*180/pi, chi*180/pi, P1bcross);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Cross-polarized Dihedral Corner Reflector in
Background','FontSize',20)

%% Problem 2

chi = -pi/4:.01:pi/4;
psi = 0:0.01:pi;
[Chi, Psi] = meshgrid(chi,psi);
I = 1;

a = 0.5;
b = 1;
S = [a, 0; 0, b];
Shh = a;
Shv = 0;
Svh = 0;
Svv = b;
M2a=[1/4*(Shh*conj(Shh)+Svv*conj(Svv)+2*Shv*conj(Shv)), ... % 1,1
    1/4 *(Shh*conj(Shh)-Svv*conj(Svv)),... % 1,2
    1/2*(real(conj(Shh)*Shv)+real(conj(Shv)*Svv)),... %1,3
    1/2*(imag(conj(Shh)*Shv)+real(imag(Shv)*Svv));... %1,4
    1/4*(Shh*conj(Shh)-Svv*conj(Svv)),...%2,1
```

```
    1/4*(Shh*conj(Shh)+Svv*conj(Svv)-2*Shv*conj(Shv)), ... %2,2
    1/2*(real(conj(Shh)*Shv)-real(conj(Shv)*Svv)),... % 2,3
    1/2*(imag(conj(Shh)*Shv)-real(imag(Shv)*Svv));...% 2,4
    1/2*(real(conj(Shh)*Shv)+real(conj(Shv)*Svv)),... %3,1
    1/2*(real(conj(Shh)*Shv)-real(conj(Shv)*Svv)),... % 3,2
    1/2*(Shv*conj(Shv)+real(conj(Shv)*Svv)), ... % 3,3
    1/2*imag(conj(Shh)*Svv); ... %3,4
    1/2*(imag(conj(Shh)*Shv)+real(imag(Shv)*Svv)),... %1,4
    1/2*(imag(conj(Shh)*Shv)-real(imag(Shv)*Svv)),...% 2,4
    1/2*imag(conj(Shh)*Svv), ... %3,4
    1/2*(Shv*conj(Shv) - real(conj(Shh)*Svv))... %4,4
    ];

P2a = zeros(length(chi),length(psi));
for m = 1:length(chi)
    for n = 1:length(psi)
        F = [I; I*cos(2*psi(n))*cos(2*chi(m));
I*sin(2*psi(n))*cos(2*chi(m)); I*sin(2*chi(m))];
        Forth = [F(1);-F(2);-F(3);-F(4)];
        P2a(m,n) = F'*M2a*F;
        P2across(m,n) = F'*M2a*Forth;
    end
end

figure; subplot(1,2,1);
surf(psi*180/pi, chi*180/pi, P2a);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Co-polarized Bragg scatter','FontSize',20)
subplot(1,2,2);
surf(psi*180/pi, chi*180/pi, P2across);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Cross-polarized Bragg scatter','FontSize',20)

%%
figure; subplot(1,2,1);
imagesc(psi*180/pi, chi*180/pi, P2a./P1a);
title({'Co-polarized Bragg / Co-polarized Dihedral '},'FontSize',20);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
colorbar;

subplot(1,2,2); imagesc(psi*180/pi, chi*180/pi, P2across./P1across);
title({'Cross-polarized Bragg / Cross-polarized Dihedral '},'FontSize',20);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
colorbar;

%% P2b
M2b = M2a;
M2b(1,1) = 2*M2a(1,1);
P2b = zeros(length(chi),length(psi));
```

```
for m = 1:length(chi)
    for n = 1:length(psi)
        F = [I; I*cos(2*psi(n))*cos(2*chi(m));
I*sin(2*psi(n))*cos(2*chi(m)); I*sin(2*chi(m))];
        Forth = [F(1);-F(2);-F(3);-F(4)];
        P2b(m,n) = F'*M2b*F;
        P2bcross(m,n) = F'*M2b*Forth;
    end
end
figure; subplot(1,2,1);
surf(psi*180/pi, chi*180/pi, P2b);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Co-polarized Bragg with Background','FontSize',20)


subplot(1,2,2); surf(psi*180/pi, chi*180/pi, P2bcross);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
zlabel('P','FontSize',20);
title('Cross-polarized Bragg with Background','FontSize',20)

%%
figure; subplot(1,2,1);
imagesc(psi*180/pi, chi*180/pi, P2b./P1b);
title({'Co-polarized Bragg / Co-polarized Dihedral ','(both with
background)'},'FontSize',20);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
colorbar;

subplot(1,2,2); imagesc(psi*180/pi, chi*180/pi, P2bcross./P1bcross);
title({'Cross-polarized Bragg / Cross-polarized Dihedral ','(both with
background)'},'FontSize',20);
xlabel('Orientation angle \psi','FontSize',20);
ylabel('Ellipticity angle \chi','FontSize',20);
colorbar;
```